PROPAGATION OF A TWO-DIMENSIONAL TURBULENT JET FROM A LINEAR SOURCE PLACED AT THE VERTEX OF A WEDGE

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We present results from the theoretical solution of a problem concerned with the propagation of a two-dimensional turbulent jet from a linear source placed at the vertex of a wedge. A calculation of flow into the jet of ejected air and the change in the resulting static pressure leads to a new law for decay of the axial velocity $u_m \sim x^{-n}$, where the exponent n is a function of the vertex angle of the wedge.

We consider the two-dimensional flow of a turbulent jet (Fig. 1) from a linear source placed at the vertex of a wedge of half-angle $(\pi - \gamma)$, where γ is an arbitrary angle, $\varphi_0 < \gamma < \pi$. In solving this problem we proceed from the fact that the velocity profile u/u_m in the jet is similar with respect to the length, depending only on the parameter $\xi \equiv y/b$. We seek a solution in the form $b \sim x$, $u_m \sim x^{-n}$. When the static pressure is assumed to be constant throughout the whole region of the flow (and, consequently, when no account is taken of the flow induced by the jet) the solution of this problem is well known [1]; in particular, it is known that $n = 1/2$. We solve this problem, assuming the static pressure to be variable, and we determine the value of the exponent in the power law describing the decay of the axial velocity.

The general solution for the induced flow outside the turbulent jet can be written, relative to a polar coordinate system (see Fig. 1), in the following way [2]:

$$v_r = -Ar^{-n}\cos(1-n)(\varphi-\gamma),$$
$$v_\varphi = Ar^{-n}\sin(1-n)(\varphi-\gamma). \tag{1}$$

We isolate a closed contour, enclosing a zone of intensive turbulent flow in the jet, as shown in Fig. 1. By a known theorem, the projection onto the x-axis of the total flow of momentum through a closed surface is equal to zero [3]:

$$\oint [P\mathbf{n} + \rho\mathbf{v}(\mathbf{vn})]_x\, dl = 0. \tag{2}$$

Here **n** is a unit vector along the normal to the contour (positive direction toward the interior of the contour); **v** is the velocity vector; l is a distance along the contour. For the contour shown in Fig. 1 the expression (2) may be written as follows:

$$\Big[\int_0^b (P + \rho u^2)\, dy\Big]_2 - \Big[\int_0^b [P + \rho u^2]\, dy\Big]_1 = [P\sin\varphi_0 - \rho v_x v_\varphi]_{\varphi=\varphi_0}\, dr. \tag{3}$$

The expression in the right-hand side of Eq. (3) is evaluated for $\varphi = \varphi_0$, i.e., on the boundary of the jet, and the subscripts $[\ ]_1$ and $[\ ]_2$ mean that the parameters within the square brackets are evaluated at the sections 1 and 2.

It is well known [4] that the pressure inside the turbulent region of the jet can be expressed in terms of the intensity of the transverse pulsations

$$P = P_{\varphi=\varphi_0} - \rho\overline{v'^2}. \tag{4}$$

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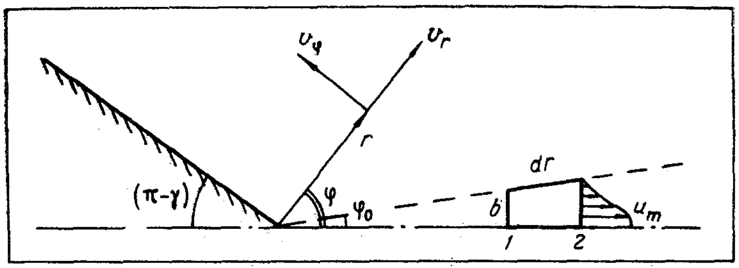

Fig. 1. Schematic representation of the two-dimensional jet.

In addition, when the term ρu^2 is averaged in relation (3), an additional term arises, with a longitudinal pulsating velocity $\rho \overline{u}'^2$.

From the condition of self-similar flow the magnitude of the pulsations can be represented in the form

$$\overline{v'^2} - \overline{u'^2} = u_m^2 \varphi(\xi), \tag{5}$$

and the mean velocity in the longitudinal direction is

$$\overline{u}^2 = u_m^2 f^2(\xi). \tag{6}$$

The magnitude of the pressure on the boundary of the jet (for $\varphi = \varphi_0$) can be obtained from the Bernoulli integral

$$P_{\varphi=\varphi_0} = P_\infty - \frac{\rho(v_r^2 + v_\varphi^2)}{2}, \tag{7}$$

where the velocities v_r and v_φ of flow into the jet may be determined for $\varphi = \varphi_0$ from the relations (1). Finally, the projection onto the x-axis of the velocity of flow of air into the jet, which enters into the relation (3), satisfies the obvious relation

$$v_x = v_r \cos\varphi_0 - v_\varphi \sin\varphi_0. \tag{8}$$

The relations (1), (4)-(8) enable us to express all the parameters in Eq. (3) in terms of characteristics of the jet itself and to simplify the integrals appearing within the square brackets. If we transform Eq. (3) in this way and then let $dr \to 0$, and then integrate the differential equation obtained through this limiting process, we obtain

$$K_1 \rho u_m^2 b = C_1 + \frac{\rho A^2 r^{1-2n}}{2} \sin\varphi_0 + \frac{\rho A^2 r^{1-2n}}{2(1-2n)} \sin\left[(1-2n)(\varphi_0 - \gamma) - \gamma\right], \tag{9}$$

where $K_1 = \int_0^1 f^2 d\xi - \int_0^1 \varphi d\xi$. From the flow conservation condition for this contour, we have

$$\left[\int_0^b \rho u dy\right]_2 - \left[\int_0^b \rho u dy\right]_1 = -\rho v_\varphi dr \tag{10}$$

and from analogous transformations we obtain a second relation

$$K_2 \rho u_m b = C_2 - \frac{\rho A r^{1-n}}{1-n} \sin(1-n)(\varphi_0 - \gamma), \tag{11}$$

where $K_2 = \int_0^1 f(\xi) d\xi$. Relations (9) and (11) furnish a general solution of the stated problem. Since we seek a solution in the form $b \sim r$ and $u_m \sim r^{-n}$, then, as it is easy to show, the integration constants C_1 and C_2 are equal to zero. From this, using the relations (9) and (11), we find the axial velocity and the thickness of the jet:

$$u_m = -\frac{Ar^{-n}}{2} \frac{K_2}{K_1} \frac{\sin\varphi_0 + \dfrac{\sin\left[(1-2n)(\varphi_0 - \gamma) - \gamma\right]}{1-2n}}{\dfrac{\sin(1-n)(\varphi_0 - \gamma)}{1-n}}, \tag{12}$$

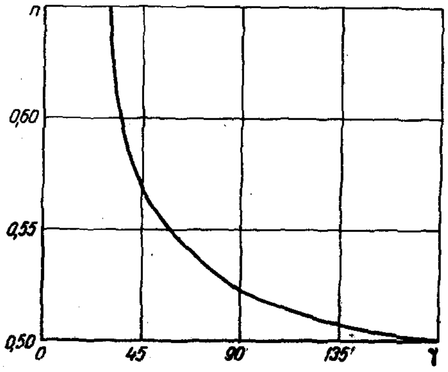

Fig. 2. Influence of the angle γ on the magnitude of the exponent n in the axial velocity decay law.

$$b = r\frac{2K_1}{K_2^2}\,\frac{\left[\dfrac{\sin(1-n)(\varphi_0-\gamma)}{1-n}\right]^2}{\sin\varphi_0+\dfrac{\sin\left[(1-2n)(\varphi_0-\gamma)-\gamma\right]}{1-2n}}. \qquad (13)$$

In these relations the constants $K_1 \approx 0.31$, $K_2 \approx 0.45$ depend on the form of the velocity profile in the jet, the values given here for them being in accord with the data from [1, 4]. The constant A is determined from the flow conditions (by the initial momentum of the jet). Before determining the value of the constants φ_0 and n, we consider some general properties of the relations (12) and (13). Assume that $\gamma \neq \pi$, i.e., that the jet flow originates, for example, from a wall ($\gamma = \pi/2$). Then the known solution with $n = 1/2$ (see [1]) does not satisfy the formulas (12) and (13). In fact, for $\gamma \neq \pi$ and $n = 1/2$, according to the relations (12) and (13), $u_m \to \infty$ and $b \to 0$ for arbitrary values of r, which physically does not make sense. Hence when we take variation of the static pressure into account, we see that for $\gamma \neq \pi$ the exponent in the axial velocity decay law cannot be equal to 1/2. To find the numerical value of n we make the fairly crude assumption that $\varphi_0 = \text{const} \approx 12.5°$. As a basis for this assumption we cite the experimental data [1] supporting the fact that the expansion angle of the submerged jet and that of the jet propagating in the oncoming flow are identical. The flow of the jet from the body, shown in Fig. 1, changes gradually, as the angle γ changes from π to φ_0, from the flow of the submerged jet to a jet in the oncoming flow, which is ejected by this jet [2]. If we make this assumption, then $b/r = \sin\varphi_0 = \text{const}$, and it then follows from Eq. (13) that

$$\frac{\sin\left[(1-2n)(\varphi_0-\gamma)-\gamma\right]}{1-2n} = \frac{2K_1}{\sin\varphi_0 K_2^2}\left[\frac{\sin(1-n)(\varphi_0-\gamma)}{1-n}\right]^2 - \sin\varphi_0. \qquad (14)$$

This is a transcendental equation relating the exponent n and the wedge angle γ. In solving it we use the following values of the constants: $\varphi_0 = 12.5°$, $2K_1/\sin\varphi_0 K_2^2 = 14.5$. The results of the calculations for $n(\gamma)$ are shown in Fig. 2. We remark that for $30° < \gamma \leq 180°$ the value of n differs little from 1/2 and Eq. (14) can be simplified by expanding the function in a series in the small parameter $(1-2n)$; from formula (14) we then obtain

$$1-2n = \frac{-\sin\gamma}{58\sin^2\dfrac{\varphi_0-\gamma}{2}-\sin\varphi_0}. \qquad (15)$$

As we have already remarked, it is evident from formula (15) and the data shown in Fig. 2 that the exponent $n = 1/2$ only when $\gamma = \pi$, and then when the angle γ decreases the magnitude of n increases, for example, when $\gamma \approx 22.5°$ the jet damping exponent $n = 1$, and as $\gamma \to \varphi_0$, the value of $n \to \infty$. This region of γ values is of no practical singificance, since for these values of γ the flow picture assumed in the calculations does not materialize. In actuality, when $\gamma \leq \varphi_0$, we have the usual flow in a two-dimensional diffusor with $n = 1$.

In concluding, we calculate formally the excess momentum of the jet, assuming that the pressure throughout the flow region is constant (which is what is usually done in the theory of submerged jets [1]): we then obtain

$$\Delta I = K_1\rho u_m^2 b = \frac{\rho A^2 r^{1-2n}}{2}\left\{\sin\varphi_0 + \frac{\sin\left[(1-2n)(\varphi_0-\gamma)-\gamma\right]}{1-2n}\right\} \approx 29\rho A^2 r^{1-2n}\sin\frac{\varphi_0-\gamma}{2}. \qquad (16)$$

It is evident that this excess momentum for $\nu \neq \pi$ is variable along the jet and that it decreases as the distance from the source increases, and for $\gamma = \pi$ and $n = 1/2$ the value $\Delta I = \text{const}$. This result differs essentially from the data given in [5], where the author first tried to take into account the additional momentum flowing into the jet. This difference is apparently due to the incorrect choice of the contour of integration, which in [5] passes through the singular point (source of the jet).

For $r \to 0$ and $n \neq 1/2$ in relation (16) the excess momentum becomes infinite. This is associated with the singularities arising from the way the problem was formulated (the jet was represented as a linear source), resulting in the fact that as $r \to 0$ the velocity in the jet and the velocity induced in the neighboring flow increases without bound, so that the pressure, in accord with the Bernoulli equation (7), may become negative. To avoid this drawback in the theory it is necessary to consider a jet of finite size at the initial section. This has the effect, finally, of changing somewhat the quantitative results obtained in this paper; it does not, however, change the qualitative result concerning the increase in the axial velocity decay exponent n as the angle γ decreases.

NOTATION

u	is the velocity;
u_m	is the velocity on jet axis;
b	is the half-width of jet;
r, φ	are the polar coordinates;
$\pi - \gamma$	is the half-angle at wedge tip;
x, y	are the Cartesian coordinates;
$\xi = y/b$	is the similarity number;
n	is the power exponent in the law $u_m \sim x^{-n}$;
v_r, v_φ	are the velocity components in polar set of coordinates r, φ;
A	is the constant;
P	is the pressure;
ρ	is the density;
$f(\xi)$, $\varphi(\xi)$	are the similarity functions;
K_1, K_2	are the constants;
C_1, C_2	are the integration constants;
ΔI	is the excess pulse of jet.

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